8^{th} Exercise sheet for Advanced Quantum Mechanics in WS16

Time-dependent perturbation theory

Exercise 17 *One-dimensional oscillator in a time-dependent electric field*

A one-dimensional harmonic oscillator with mass m, frequency ω and electric charge e moves along the positive x–axis. For $t < 0$ the harmonic oscillator is in its ground state and in the time period $0 < t < t_0$ the oscillator is influenced by a time-dependent electric field

$$E = E_0 \sin(kx - \omega_0 t), \tag{1}$$

which is oriented along the positive x–axis.

1. Calculate the transition amplitude in case of a transition in the first excited state to first order in time-dependent perturbation theory

2. Show that the transition amplitude is resonant for a certain frequency ω_0. Calculate the transition probability and the order of magnitude of the width of the resonance peak.

Exercise 18 *One-dimensional harmonic oscillator in a time-dependent potential*

A one-dimensional harmonic oscillator of mass m and frequency ω is subject to the following time-dependent perturbation potential:

$$H_1 = A \, \frac{\alpha^3 x^3 - 1}{1 + \left(\frac{t}{\tau}\right)^2}\,, \tag{2}$$

where $\alpha = \sqrt{m\omega/\hbar}$ and τ is a real parameter.

1. Assuming that for $t \to -\infty$ the oscillator is in the fundamental state, determine all the possible excited states which are allowed in first-order perturbation theory.

2. Calculate the probability of finding the oscillator in the first excited state for $t \to \infty$.

Exercise 19 *Trapped electron with time-dependent magnetic field*

Let an electron be trapped in a rigid spherical cavity of radius R, i.e. in a spherical potential of the form

$$V(\boldsymbol{r}) = \begin{cases} 0 & \forall\, r < R \\ \infty & \forall\, r \geq R \end{cases}, \qquad r = |\boldsymbol{r}|. \tag{3}$$

1. Solve the stationary Schrödinger equation for potential $V(\boldsymbol{r})$ and calculate the energy of the states $l = 0, 1$. Neglect the spin of the electron.

2. Consider the electron in the state $l = 0$. We now apply a constant homogeneous magnetic field B along the z axis and also prepare the electron spin so that, at $t = 0$, the spin is an eigenstate of S_x with positive eigenvalue ("is along the positive x-axis"). Calculate the probability of finding the spin along the y axis as a function of time.

3. We finally add another magnetic field B' rotating in the x-y plane with frequency ω, with components

   $$B'_x = B' \cos \omega t, \qquad B'_y = B' \sin \omega t. \tag{4}$$

   Write down the Schrödinger equation for the spinor of the electron and solve it as a function of time, given that at $t = 0$ the spin of the electron is along the z-axis.