

7th Exercise sheet for Advanced Quantum Mechanics in WS16

Exercise 16 *Two fermions in a one-dimensional harmonic oscillator*

Consider a one-dimensional system of two spin-1/2 particles of mass μ , whose dynamics is described by the Hamiltonian

$$H_0 = \frac{1}{2\mu} (p_1^2 + p_2^2) + \frac{1}{2}\mu\omega^2 (x_1^2 + x_2^2). \quad (1)$$

- a) Calculate the eigenvalues and wave functions of the ground state and the first excited state, and determine their degree of degeneracy.

For the following exercises H_0 will be extended by the perturbation potential $H_1 = 4gx_1x_2\mathbf{S}_1 \cdot \mathbf{S}_2$, where \mathbf{S}_1 and \mathbf{S}_2 describe the spin of both particles.

- b) Calculate the corrections to the ground state and the first excited state to first order ($\mathcal{O}(g)$) in perturbation theory.
- c) Calculate the exact ground state and first excited state and their eigenvalues of the full Hamiltonian, i.e. $H = H_0 + H_1$, and compare the results with those you obtained in b).