6^{th} Exercise sheet for Advanced Quantum Mechanics in WS16

Exercise 14 *Energy eigenstates, their quantum numbers, and interactions*

The dynamics of an electron shall be given by the Hamiltonian $H = H_0 + H_1$, with

$$H_0 = \frac{\boldsymbol{p}^2}{2\mu} - \frac{e^2}{|\boldsymbol{r}|}, \qquad H_1 = 2\lambda \frac{\boldsymbol{S} \cdot \boldsymbol{r}}{|\boldsymbol{r}|}, \tag{1}$$

where μ is the reduced mass and $\boldsymbol{S} = \frac{1}{2}\boldsymbol{\sigma}$ denotes the spin of the electron.

a) Are energy eigenstates of the operator H also parity eigenstates?

b) Which of the following operators correspond to observables that are conserved: the angular momentum squared $\boldsymbol{L}^2$, the spin $\boldsymbol{S}$, its square $\boldsymbol{S}^2$, the sum of angular momenta $\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$ and/or $\boldsymbol{J}^2$? Compare this case with the hydrogen atom, i.e. the case where $H_1 = 0$.

Exercise 15 *Spherical nucleus in a hydrogen-like atom*

To calculate the spectrum of a hydrogen-like atom the approximation is made that the charge of its nucleus is concentrated in a point. In this exercise we would like to improve this model, and instead assume the nucleus to be a ball of radius R in which the total charge Ze is homogeneously distributed.

a) Calculate the potential of the nucleus that acts on the electron. Sketch the potential and compare it with the potential of a point charge.

b) Under which condition can we treat the potential derived in a) with perturbation theory, i.e. what is the parameter that must be small?

c) Use perturbation theory to calculate the first-order shift of the ground state.

d) Derive the sign of the shift again, this time without the calculation in c).